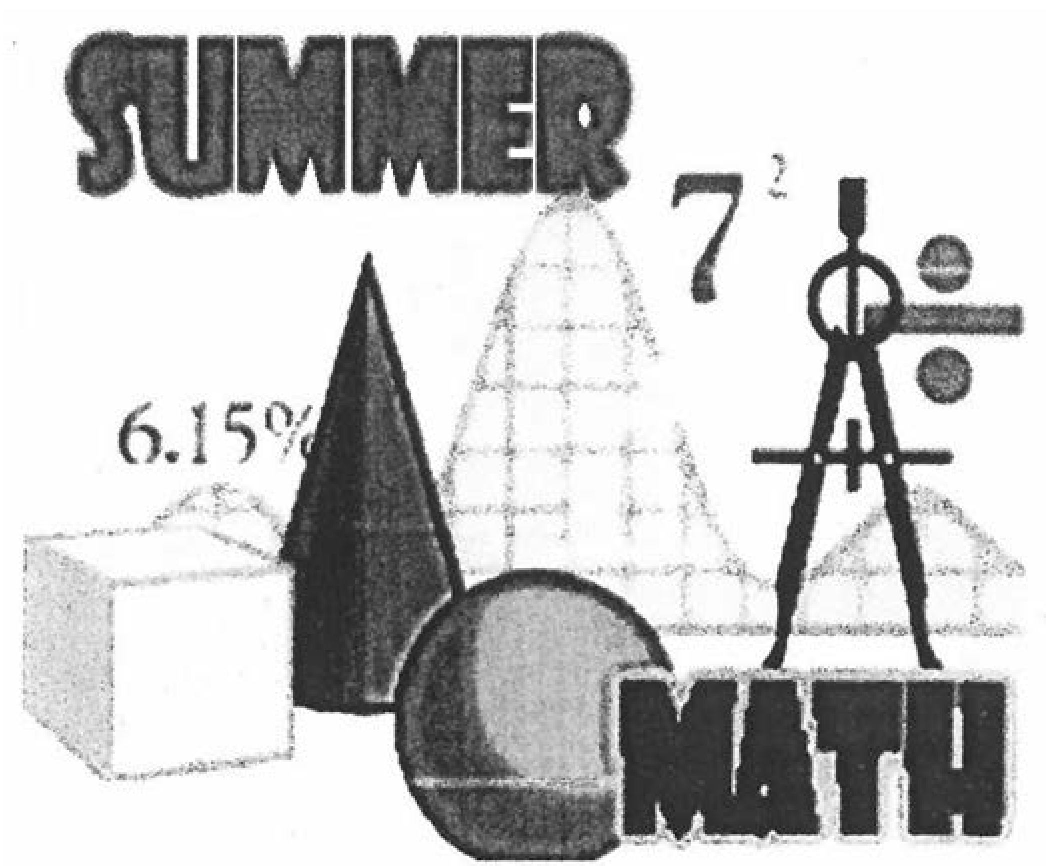
Grade 8

Please show all work. Do not use a calculator!

Please refer to reference section and examples.



Name ______________________

Date due: Tuesday September 4, 2018

June 2018

Dear Middle School Parents,

After the positive feedback we received from last year's summer math program, we decided to continue with it this year. Practicing math skills will tremendously help students in throughout the year. It is important that the students do not experience loss over the summer. Please also, have your child continue to practice basis math facts as needed.

The math packets are to be completed over the summer. The packets are designed as review of previously learned topics.

It is highly recommended that your child complete a portion of the packet each week, so they do not feel overwhelmed. (1 hour each week, suggested time 3 days for 20 minutes)
The packet will be due September 4, 2018 and will be the first test grade in your child's math class. There will be a penalty for late work.

Students need to show all work and should not use a calculator!

We strive to help our students meet high academic success.

Sincerely,
Mrs. Doucette
Mrs. Reid

Cc: Mrs. Sullivan

My child ________________________________has completed their Summer Math Packet.

____
Parent Signature

ANSWER SHEET

1	26	51	76
2	27	52	77
3	28	53	78
4	29	54	79
5	30	55	80
6	31	56	81
7	32	57	82
8	33	58	83
9	34	59	84
10	35	60	85
11	36	61	86
12	37	62	87
13	38	63	88
14	39	64	89
15	40	65	90
16	41	66	91
17	42	67	92
18	43	68	93
19	44	69	94
20	45	70	95
21	46	71	96
22	47	72	97
23	48	73	98
24	49	74	99
25	50	75	100

ANSWER SHEET

101	126	151	
102	127	152	
103	128	153	
104	129	154	
105	130		
106	131		
107	132		
108	133		
109	134		
110	135		
111	136		
112	137		
113	138		
114	139		
115	140		
116	141		
117	142		
118	143		
119	144		
120	145		
121	146		
122	147		
123	148		
124	149		
125	150		

Grade 8 Name___________________________________

Summer Math

Mrs. Doucette and Mrs. Reid______

Evaluate each expression. Add or Subtract

1) $0.9 - 1.2$

2) $0.1 - 6.3$

3) $(-5.6) + 5.6$

4) $(-6.5) + (-2.5)$

5) $1\frac{1}{3} + \left(-\frac{2}{7}\right)$

6) $\left(-3\frac{5}{8}\right) + \left(-1\frac{1}{2}\right)$

7) $(-4) - 7$

8) $(-2) + (-2)$

9) $(-1) + 7$

10) $(-7) - (-5)$

11) $4\frac{3}{4} + \left(-2\frac{3}{5}\right)$

12) $\frac{1}{5} - 2\frac{2}{3}$

13) $28 + (-27) - 35$

14) $49 + (-47) - 9$

Simplify each expression. Combine like terms, use the distributive property when necessary.

15) $-3n - 4n$

16) $-9p + 7 + 1$

17) $x + 2 + 5$

18) $7x + 9 - 2x$

19) $-2(3v + 9)$

20) $-2(7 + 3p)$

21) $r + 5(10 + 3r)$

22) $2(p - 1) + 2$

23) $-8(r + 10) + 9$

24) $-8(p - 4) + 6$

Write each as a fraction.

25) 0.005

26) 0.345

27) $0.\overline{72}$

28) 0.9

Write each as a percent. Round to the nearest tenth of a percent.

29) 3.398

30) 0.006

31) 1.62

32) 0.03

Find each product.

33) -3×0.8

34) -3.5×-5.3

35) 9.5×-2.8

36) 8.5×-1.8

Find each quotient. Round to the nearest hundredth.

37) $-8 \div 2.5$

38) $-9 \div 3.4$

39) $-4.7 \div -8.9$

40) $5.4 \div 6.5$

Find each quotient.

41) $1 \div \frac{19}{10}$

42) $\frac{3}{10} \div \frac{-3}{5}$

43) $\frac{5}{3} \div \frac{1}{4}$

44) $-2 \div \frac{14}{9}$

45) $-2\frac{1}{2} \div 1\frac{2}{7}$

46) $2\frac{4}{7} \div 2\frac{1}{2}$

Solve each equation.

47) $31 = x - (-6)$

48) $-40 = r - 12$

49) $\frac{k}{25} = \frac{26}{25}$

50) $21v = -105$

51) $7 = 5 + \frac{x}{6}$

52) $-15 = -10 - 5x$

53) $-3 = \frac{x}{1} - 3$

54) $\frac{a + 3}{4} = -1$

55) $166 = 3(1 + 8x) - 5$

56) $-301 = -7(1 - 7x)$

57) $181 = -8(5v + 8) + 5$

58) $8(4b - 3) + 4 = -116$

59) On Tuesday Ryan bought five posters. On Wednesday half of all the posters that he had were destroyed. On Thursday there were only 21 left. How many did he have on Monday?

60) Stephanie spent half of her weekly allowance buying pizza. To earn more money her parents let her clean the oven for $4. What is her weekly allowance if she ended with $12?

61) Maria had $20 to spend on seven advocados. After buying them she had $6. How much did each advocado cost?

62) A wise man once said, "400 reduced by 3 times my age is 100." What is his age?

Evaluate each using the values given.

63) $z - (y - 4)$; use $y = 5$, and $z = 2$

64) $x(y + y)$; use $x = 5$, and $y = 2$

65) $b + a^2$; use $a = 1$, and $b = 1$

66) $(x + y) \div 2$; use $x = 6$, and $y = 4$

67) $a(b + c)$; use $a = 2\frac{3}{4}$, $b = 2\frac{5}{6}$, and $c = 1\frac{1}{6}$

68) $j - (h + h)$; use $h = 1\frac{2}{5}$, and $j = 3\frac{1}{2}$

Simplify. Your answer should contain only positive exponents.

69) $7n^4 \cdot 4n^3$

70) $x^4 \cdot 6x^3$

71) $5p \cdot 3p \cdot 8p^4$

72) $2n^3 \cdot 5n^4 \cdot n^4$

73) $\dfrac{4v^2}{2v}$

74) $\dfrac{4k^2}{5k^2}$

75) $\dfrac{8x}{4x}$

76) $\dfrac{7n^2}{4n^3}$

77) $7x \cdot 6x^3$

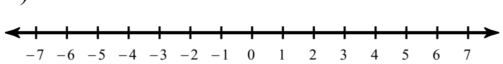
78) $p^4 \cdot 8p^3$

79) $8a^4 \cdot 4a^4$

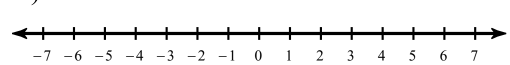
80) kk^2

Draw a graph for each inequality.

81) $b \leq 2$

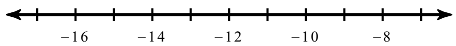


82) $x > 1$

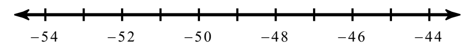


Solve each inequality and graph its solution.

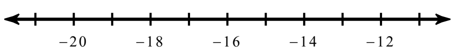
83) $-14 > n - 2$



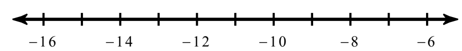
84) $\frac{x}{8} \le -6$



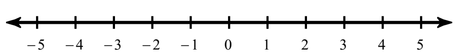
85) $81 > -6k - 3$



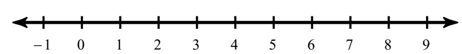
86) $-1 \le \frac{8 + n}{6}$



87) $-x + 8(6x + 3) < 118$



88) $-3(-7n - 6) > 165$



Solve each problem.

89) 77 is 80% of what?

90) 31% of 95 is what?

91) 5 is what percent of 27.1?

92) 47 is what percent of 65.8?

93) 74% of 16 is what?

94) What percent of 49 is 39?

Evaluate each expression.

95) $(-6) + 10 + 6 - (-5)$

96) $-\dfrac{14}{1 - 3} \times 2$

97) $(-2) - (-9)^2 - 2$

98) $(-2)((-7) + (-9) - (-8))$

Simplify each expression.

99) $4 - 3(1 + 3k)$

100) $8(6 + 5x) + 8$

101) $-7(5 + 2n) - 6$

102) $-7(1 + 3x) + 1$

Write each as an algebraic expression.

103) twice a number is equal to 10

104) the quotient of 70 and 7

105) the quotient of 50 and a number

106) v minus 14

107) half of 14

108) the sum of n and 12 is 33

109) the sum of w and 8 is greater than or equal to 45

110) 9 less than n is greater than 14

111) n cubed is greater than 33

112) the difference of x and 18 is greater than 29

Write the prime factorization of each. Do not use exponents.

113) 44

114) 47

115) 56

116) 75

117) $15b$

118) $48m^2$

119) $52x^2$

120) $45mn$

Simplify. Your answer should contain only positive exponents.

121) x^4y^{-4}

122) $4y^{-3}$

123) $7y^{-1}$

124) $4x^{-2}$

125) $\left(\frac{4n^2}{3n}\right)^2$

126) $\left(\frac{m}{3m}\right)^3$

127) $\frac{2m^2}{\left(2m^3\right)^3}$

128) $\left(\frac{r^4}{4r}\right)^3$

Solve each proportion.

129) $\frac{a}{8} = \frac{7}{2}$

130) $\frac{5}{7} = \frac{n}{3}$

131) $\frac{2}{3} = \frac{5}{r}$

132) $\frac{n}{3} = \frac{6}{5}$

Find the GCF of each.

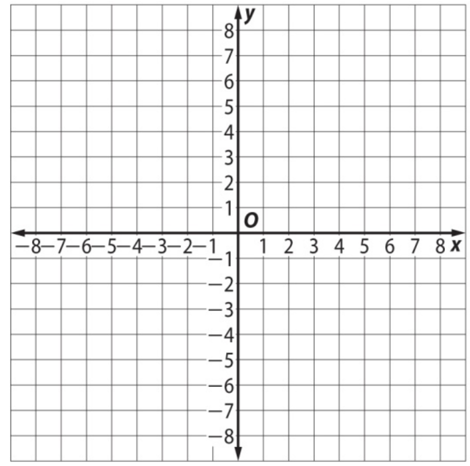
133) $14x^3, 42x^2y$

134) $18y^2x, 45x^3y$

135) $24xy, 48y^3$

136) $36m^4, 48m^2$

Graph and label each point on the coordinate plane.
Name the quadrant in which each point is located.



137. $A(2, 6)$

138. $B(-1, 4)$

139. $C(0, -5)$

140. $D(-4, -3)$

141. $E(2, 0)$

142. $F(3, -2)$

143. $G(-4, 4)$

144. $H(2, -5)$

145. $I(6, 3)$

146. $J(-5, -8)$

147. $K(3, -5)$

148. $L(-7, -3)$

Solve. Check your solutions.

149. $8(g - 3) = 24$

150. $5(x + 3) = 25$

151. $2(3d + 7) = 5 + 6d$

152. $2(s + 11) = 5(s + 2)$

153. $7y - 1 = 2(y + 3) - 2$

154. $2(f + 3) - 2 = 8 + 2f$

Words and Expressions

A **numerical expression** contains a combination of numbers and operations such as addition, subtraction, multiplication, and division. Verbal phrases can be translated into numerical expressions by replacing words with operations and numbers.

+	−	×	÷
plus	minus	times	divide
the sum of	the difference of	the product of	the quotient of
increased by	decreased by	of	divided by
more than	less than		among

Example 1: Write a numerical expression for the verbal phrase.

the product of seventeen and three

Phrase the **product** of seventeen and three

Expression 17×3

Evaluate, or find the numerical value of, expressions with more than one operation by following the **order of operations**.

Step 1 Evaluate the expressions inside grouping symbols.

Step 2 Multiply and/or divide from left to right.

Step 3 Add and/or subtract from left to right.

Example 2 Evaluate the expression.

$4(3 + 6) + 2 \cdot 11$

$4(3 + 6) + 2 \cdot 11 = 4(9) + 2 \cdot 11$ Evaluate (3 + 6).

$= 36 + 22$ Multiply 4 and 9, and 2 and 11.

$= 58$ Add 36 and 22.

Variables and Expressions

An **algebraic expression** is a combination of variables, numbers, and at least one operation. A **variable** is a letter or symbol used to represent an unknown value. To translate verbal phrases with an unknown quantity into algebraic expressions, first define the variable.

Algebraic Expressions

The letter *x* is most often used as a variable. → $x + 3c$

$7d$ means $7 \times d$. → $7d - 2$

mn means $m \times n$. → mn

$\frac{b}{5}$ means $b \div 5$. → $\frac{b}{5}$

To evaluate an algebraic expression, replace the variable(s) with known values and follow the order of operations.

Substitution Property of Equality

Words If two quantities are equal, then one quantity can be replaced by the other.

Symbols For all numbers a and b, if $a = b$, then a may be replaced by b.

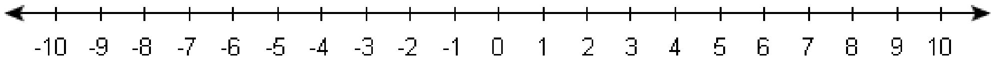
Example: Evaluate the expression if $r = 6$ and $s = 2$.

$8s - 2r$

$8s - 2r = 8(2) - 2(6)$ Replace *r* with 6 and *s* with 2.

$= 16 - 12$ or 4 Multiply. Then subtract.

Topic: Integers



Examples:

Addition	Subtraction	Multiplication	Division
Same signs: *Add & keep sign* $^{+}6 + {}^{+}5 = {}^{+}11$ $^{-}8 + {}^{-}2 = {}^{-}10$	*Keep–Change-Opposite* $^{+}10 - {}^{-}8 = {}^{+}10 + {}^{+}8 = 18$ $^{-}5 - {}^{+}12 = {}^{-}5 + {}^{-}12$ $^{-}20 - {}^{-}8 = {}^{-}20 + {}^{-}8 = {}^{-}12$	*Same signs:* *Positive product* $(^{+}7)(^{+}8) = {}^{+}56$ $(^{-}2)(^{-}6) = {}^{+}12$	*Same signs:* *Positive quotient* $^{+}42 / {}^{+}6 = {}^{+}7$ $^{-}24 / {}^{-}8 = {}^{+}3$
Different signs: *Subtract & take sign of larger value* $^{+}9 + {}^{-}5 = {}^{+}4$ $^{-}6 + {}^{+}1 = {}^{-}5$		*Different signs:* *Negative product* $(^{+}3)(^{-}9) = {}^{-}27$ $(^{-}5)(^{+}4) = {}^{-}20$	*Different signs:* *Negative quotient* $^{+}56 / {}^{-}7 = {}^{-}8$ $^{-}50 / {}^{+}2 = {}^{-}25$

Recall the **order of operations**:

1 – **P**arentheses (or grouping symbols)
2 - **E**xponents
3 - **M**ultiplication / **D**ivision (left to right)
4 - **A**ddition/**S**ubtraction (left to right)

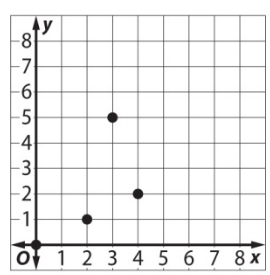
Ordered Pairs and Relations

In mathematics, a **coordinate system** or **coordinate plane** is used to locate points. The horizontal number line is called the ***x*-axis** and the vertical number line is called the ***y*-axis**. The point where the two axes intersect is the **origin** (0, 0). An **ordered pair** of numbers is used to locate points in the coordinate plane. The point (4, 3) has an ***x*-coordinate** of 4 and a ***y*-coordinate** of 3.

A **relation** is a set of ordered pairs, such as {(0, 3), (1, 2), (3, 6), (7, 4)}. A relation can also be shown in a table or a graph. The set of *x*-coordinates is the **domain** of the relation, while the set of *y*-coordinates is the **range** of the relation.

Example: Express the relation {(0, 0), (2, 1), (4, 2), (3, 5)} as a table and as a graph. Then determine the domain and range.

x	*y*
0	0
2	1
4	2
3	5



The domain is {0, 2, 4, 3}, and the range is {0, 1, 2, 5}.

Integers and Absolute Value

A **negative number** is a number less than zero. A **positive number** is a number greater than zero. The set of **integers** can be written {…, –3, –2, –1, 0, 1, 2, 3, …} where … means *continues indefinitely*. Two integers can be compared using an **inequality**, which is a mathematical sentence containing < or >.

Example 1: Write an integer for each situation. Then identify its opposite and describe what it means.

a. 16 feet below the surface

The integer is –16.
The opposite is 16.
It means 16 feet above the surface.

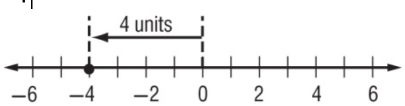
b. 5 strokes over par

The integer is +5 or 5.
The opposite is –5.
It means 5 strokes below pa

Numbers on opposite sides of zero and the same distance from zero have the same **absolute value**. The symbol for absolute value is two vertical bars on either side of the number. |2| = 2 and |–2| = 2

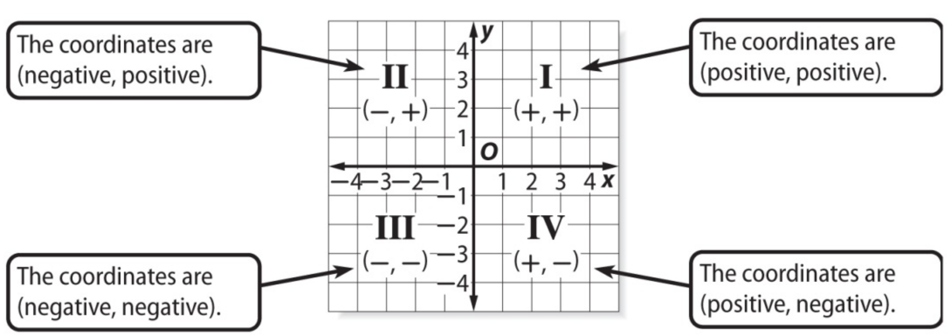
Example 2: Evaluate each expression.

a. |–4|



|–4| = 4 On the number line, –4 is 4 units from 0.

Graphing in Four Quadrants



Example 1: Graph and label each point on a coordinate plane. Name the quadrant in which each point lies.

a. ***M*(– 2, 5)**

Start at the origin. Move 2 units left.

Then move 5 units up and draw a dot.

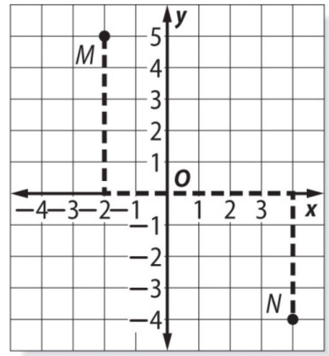
Point *M*(–2, 5) is in Quadrant II.

b. ***N*(4, –4)**

Start at the origin. Move 4 units right.

Then move 4 units down and draw a dot.

Point $N(4, -4)$ is in Quadrant IV.



Fractions and Decimals

Some fractions can be written as decimals by making equivalent fractions with denominators of 10, 100, or 1,000. All fractions can be written as decimals by dividing the numerator by the denominator. **Repeating decimals** have a pattern in their digits that repeats without ending. If the repeating digit is zero, then the decimal is a **terminating decimal.**

Example 1 **Write $\frac{3}{8}$ as a decimal.**

$\frac{3}{8} \longrightarrow 0.375$

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

0.375 is a terminating decimal.

Example 2 **Write $\frac{4}{9}$ as a decimal.**

$\frac{4}{9} \longrightarrow 0.444$

$$\begin{array}{r} 0.444 \\ 9\overline{)4.000} \\ \underline{3\,6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

0.444... is a repeating decimal. You can indicate that a decimal repeats by writing a bar or line over the repeating digit(s): $\frac{4}{9} = 0.\overline{4}$.

It may be easier to compare numbers when they are written as decimals.

Multiplying Rational Numbers

To multiply fractions, multiply the numerators and multiply the denominators: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, where b, $d \neq 0$. Fractions may be simplified before or after multiplying. For negative fractions, assign the negative sign to the numerator.

Example 1: **Find $7\frac{1}{2} \cdot \frac{2}{3}$. Write in simplest form.**

$7\frac{1}{2} \cdot 2\frac{2}{3} = \frac{15}{2} \cdot \frac{8}{3}$ Rename mixed numbers as improper fractions.

$= \frac{\overset{5}{\cancel{15}}}{\cancel{2}} \cdot \frac{\overset{4}{\cancel{8}}}{\cancel{3}}$ Divide 15 and 3 by 3, and 8 and 2 by 2.

$= \frac{5 \cdot 4}{1 \cdot 1}$ Multiply.

$= \frac{20}{1}$ or 20 Simplify.

Algebraic expressions are expressions which contain one or more variables. Variables can represent fractions in algebraic expressions.

Example 2: Evaluate $\frac{2}{3}ab$ if $a = 3\frac{3}{7}$ and $b = -\frac{5}{12}$. Write the product in simplest form.

$\frac{2}{3}ab = \frac{2}{3}\left(3\frac{3}{7}\right)\left(-\frac{5}{12}\right)$ Replace a with $3\frac{3}{7}$ and b with $-\frac{5}{12}$.

$= \frac{2}{3}\left(\frac{24}{7}\right)\left(\frac{-5}{12}\right)$ Rename $3\frac{3}{7}$ as $\frac{24}{7}$.

$= \frac{2}{3}\left(\frac{\overset{2}{\cancel{24}}}{7}\right)\left(\frac{-5}{\underset{1}{\cancel{12}}}\right)$ The GCF of 24 and 12 is 12.

$= -\frac{20}{21}$ or $-\frac{20}{21}$ Simplify.

Dividing Rational Numbers

Two numbers whose product is 1 are called multiplicative inverses or reciprocals.
For any fraction $\frac{a}{b}$, where $a, b \neq 0$, $\frac{b}{a}$ is the multiplicative inverse and $\frac{a}{b} \cdot \frac{b}{a} = 1$.
This means that $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses because $\frac{2}{3} \cdot \frac{3}{2} = 1$.
To divide by a fraction, multiply by its multiplicative inverse: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, where $b, c, d \neq 0$.

Example 1: Find $\frac{3}{4} \div \frac{5}{8}$. Write in simplest form.

$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \cdot \frac{8}{5}$ Multiply by the multiplicative inverse of $\frac{5}{8}$, $\frac{8}{5}$.

$= \frac{3}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{5}$ Divide 4 and 8 by their GCF, 4.

$= \frac{6}{5}$ or $1\frac{1}{5}$ Simplify.

Algebraic fractions are fractions which contain one or more variables. You can divide algebraic fractions just as you would divide numerical fractions.

Example 2: Find $\frac{4}{qrs} \div \frac{10}{qs}$. Write in simplest form.

$\frac{4}{qrs} \div \frac{10}{qs} = \frac{4}{qrs} \cdot \frac{qs}{10}$ Multiply by the reciprocal of $\frac{10}{qs}$, $\frac{qs}{10}$.

$= \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{q}}r\underset{1}{\cancel{s}}} \cdot \frac{\overset{1}{\cancel{qs}}}{\underset{5}{\cancel{10}}}$ Divide out common factors.

$= \frac{2}{5r}$ Simplify.

Adding and Subtracting Unlike Fractions

Fractions with different denominators are called **unlike fractions.** To add fractions with unlike denominators, rename the fractions with a common denominator. Then add and simplify.

Example 1: Find $\frac{4}{7} + \frac{1}{3}$. Write in simplest form.

$\frac{4}{7} + \frac{1}{3} = \frac{4}{7} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{7}{7}$ Use $7 \cdot 3$ or 21 as the common denominator.

$= \frac{12}{21} + \frac{7}{21}$ Rename each fraction with the common denominator.

$= \frac{19}{21}$ Add the numerators.

To subtract fractions with unlike denominators, rename the fractionns with a common denominator. Then subtract and simplify.

Example 2: **Find $9\frac{2}{9} + \frac{5}{6}$. Write in simplest form.**

$9\frac{2}{9} - 8\frac{5}{6} = \frac{83}{9} - \frac{53}{6}$ Write the mixed numbers as improper fractions.

$= \frac{83}{9} \cdot \frac{2}{2} - \frac{53}{6} \cdot \frac{3}{3}$ Rename fractions using the LCD, 18.

$= \frac{166}{18} - \frac{159}{18}$ Simplify.

$= \frac{7}{18}$ Subtract the numerators.

Negative Exponents

A **negative exponent** is the result of repeated division. Extending the pattern below shows that $4^{-1} = \frac{1}{4}$ or $\frac{1}{4^1}$.

$4^2 = 16$
÷ 4
$4^1 = 4$
÷ 4
$4^0 = 1$
÷ 4
$4^{-1} = \frac{1}{4}$

This suggests the following definition.

$a^{-n} = \frac{1}{a^n}$ for *a* ≠ 0 and any whole number *n*. Example: $6^{-4} = \frac{1}{6^4}$

For *a* ≠ 0, $a^0 = 1$. Example: $9^0 = 1$

Example 1 **Write each expression using a positive exponent.**

a. 3^{-4}

$3^{-4} = \frac{1}{3^4}$ Definition of negative exponent

b. y^{-2}

$y^{-2} = \frac{1}{y^2}$ Definition of negative exponent

Example 2

a. $\frac{1}{6^3}$

$\frac{1}{6^3} = 6^{-3}$ Definition of negative exponent

b. $\frac{1}{81}$

$\frac{1}{81} = \frac{1}{9^2}$ Definition of exponent

$= 9^{-2}$ Definition of negative exponent

Multiplying and Dividing Monomials

When multiplying powers with the same base

Symbols	$a^m \cdot a^n = a^{m+n}$
Example	$4^2 \cdot 4^5 = 4^{2+5}$ or 4^7

Example 1: Find the product $5^7 \cdot 5$.

$5^7 \cdot 5 = 5^7 \cdot 5^1$ $5 = 5^1$

$= 5^{7+1}$ Product of Powers Property; the common base is 5.

$= 5^8$ Add the exponents.

Example 2: Find the product $2a^2 \cdot 3a$.

$2a^{-2} \cdot 3a = 2 \cdot 3 \cdot a^{-2} \cdot a$ Commutative Property of Multiplication

$= 2 \cdot 3 \cdot a^{-2+1}$ Product of Powers Property; the common base is *a*.

$= 2 \cdot 3 \cdot a^{-1}$ Add the exponents.

$= 6a^{-1}$ Multiply.

When dividing powers with the same base, subtract the exponents.

Symbols	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$
Example	$\frac{5^6}{5^2} = 5^{6-2}$ or 5^4

Example 3: Find the quotient $\frac{(-8)^4}{(-8)^2}$.

$\frac{(-8)^4}{(-8)^2} = (-8)^{4-2}$ Quotient of Powers Property; the common base is (–8).

$= (-8)^2$ Subtract the exponents.

Solving Proportions

A **proportion** is an equation stating that two ratios or rates are equal.

$$\frac{a}{b} = \frac{c}{d}$$

An important property of proportions is that their cross products are equal. You can use this property to solve problems involving proportions.

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Example **Solve the proportion $\frac{14.1}{c} = \frac{3}{4}$.**

$\frac{14.1}{c} = \frac{3}{4}$

$14.1 \cdot 4 = c \cdot 3$ Cross products

$56.4 = 3c$ Multiply.

$\frac{56.4}{3} = \frac{3c}{3}$ Divide.

$18.8 = c$ Simplify.

The solution is 18.8.

Using the Percent Proportion

In a **percent proportion,** one ratio compares *part* of a quantity to the *whole* quantity. The other ratio is the equivalent percent, written as a fraction, with a denominator of 100.

Example 1: Find each percent.

a. Twelve is what percent of 16?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{12}{16} = \frac{p}{100}$ Replace the variables.

$12 \cdot 100 = p \cdot 16$ Find the cross products.

$1200 = 16p$ Multiply.

$75 = p$ Divide.

So, twelve is 75% of 16.

b. What percent of 8 is 7?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{7}{8} = \frac{p}{100}$

$p \cdot 8 = 100 \cdot 7$

$700 = 8p$

$87.5 = p$

So, 87.5% of 8 is 7.

Example 2: **Find the part or the whole.**

a. What number is 1.4% of 15?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{a}{15} = \frac{1.4}{100}$ Replace the variables.

$a \cdot 100 = 15 \cdot 1.4$ Find the cross products.

$100a = 21$ Multiply.

$a = 0.21$ Divide.

So, 0.21 is 1.4% of 15.

b. 225 is 36% of what number?

$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{225}{b} = \frac{36}{100}$

$225 \cdot 100 = 36 \cdot b$

$22{,}500 = 36b$

$625 = b$

So, 225 is 36% of 625.

Find Percent of a Number Mentally

When working with common percents like 10%, 25%, 40%, and 50%, it may be helpful to use the fraction form of the percent.

Percent-Fraction Equivalents

$25\% = \frac{1}{4}$	$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3} = \frac{1}{6}$
$50\% = \frac{1}{2}$	$30\% = \frac{3}{10}$	$40\% = \frac{2}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$33\frac{1}{3} = \frac{1}{3}$
$75\% = \frac{3}{4}$	$70\% = \frac{7}{10}$	$60\% = \frac{3}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$	$66\frac{2}{3} = \frac{2}{3}$
$100\% = 1$	$90\% = \frac{9}{10}$	$80\% = \frac{4}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3} = \frac{5}{6}$

Example 1: **Find 20% of 35 mentally.**

20% of $35 = \frac{1}{5}$ of 35 Think: $20\% = \frac{1}{5}$.

$= 7$ Think: $\frac{1}{5}$ of 35 is 7. So, 20% of 35 is 7.

When an exact answer is not needed, estimate by rounding and using mental math to compute the answer.

Example 2: **Estimate.**

a. 23% of 84

23% is about 25% or $\frac{1}{4}$.

$\frac{1}{4}$ of 84 is 21.

So, 23% of 84 is about 21.

b. $\frac{1}{2}$ **% of 490**

$\frac{1}{2}\% = \frac{1}{2} \cdot 1\%$

490 is almost 500.

So, $\frac{1}{2}\%$ of 490 is about $\frac{1}{2} \times 5$ or 2.5.

Using the Percent Equation

A **percent equation** is an equivalent form of the percent proportion. In a percent equation, the percent is written as a decimal.

Example: Solve each problem using a percent equation.

a. Find 22% of 95.

$n = 0.22(95)$

$n = 20.9$

So, 22% of 95 is 20.9.

b. 15 is what percent of 75?

$15 = n(75)$

$0.2 = n$

So, 15 is 20% of 75.

c. 90 is 20% of what number?

$90 = 0.2n$

$450 = n$

So, 90 is 20% of 450.

Simplifying Algebraic Expressions

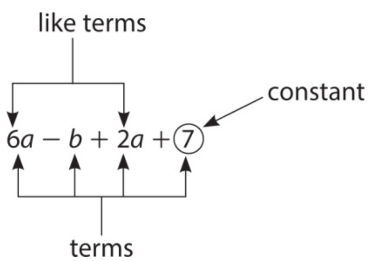
Listed below are some definitions related to algebraic expressions.

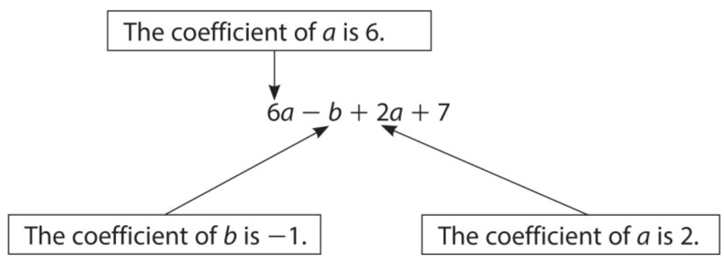
term: a number, variable, or a product of numbers and variables; terms in an expression are separated by addition or subtraction signs

coefficient: the numerical part of a term that also contains a variable

constant: term without a variable

like terms: terms that contain the same variables





When an algebraic expression has no like terms and no parentheses, it is in **simplest form.**
To make it easier to simplify an algebraic expression, rewrite subtraction as addition.
Then use the Commutative Property to group like terms together.

Example **Simplify $5t – 7(s – 4t)$.**

$5t – 7(s – 4t)$	$= 5t + (–7)[s + (–4t)]$	Definition of Subtraction
	$= 5t + (–7s) + (–7 \cdot –4t)$	Distributive Property
	$= 5t + (–7s) + 28t$	Simplify.
	$= 5t + 28t + (–7s)$	Commutative Property
	$= 33t + (–7s)$ or $33t – 7s$	Simplify.

Factoring Linear Expressions

A **linear expression** is in factored form when it is expressed as the product of its factors.

Example 1 **Factor $5x + 10$.**

Use the GCF to factor the linear expression.

$5x = 5 \cdot x$ Write the prime factorization of 5*x* and 10.

$10 = 5 \cdot 2$ Circle the common factors.

The GCF of 5*x* and 10 is 5. Write each term as a product of the GCF and its remaining factors.

$5x + 10 = 5(x) + 5(2)$

$= 5(x + 2)$ Distributive Property

So, $5x + 10 = 5(x + 2)$.

Example 2 **Factor $3x + 8$.**

$3x = 3 \cdot x$ Write the prime factorization of $3x$ and 8.

$8 = 2 \cdot 2 \cdot 2$

There are no common factors, so $3x + 8$ *cannot be factored.*

Solving Two-Step Equations

A two-step equation contains two operations. To solve two-step equations, use inverse operations to undo each operation in reverse order of the order of operations. First, undo addition/subtraction. Then, undo multiplication/division.

Example 1 **Solve $\frac{1}{2}c - 13 = 7$. Check your solution.**

$\frac{1}{2}c - 13 = 7$	Write the equation.
$\frac{1}{2}c - 13 + 13 = 7 + 13$	Addition Property of Equality
$\frac{1}{2}c = 20$	Simplify.
$2 \cdot \frac{1}{2}c = 2 \cdot 20$	Multiplication Property of Equality
$c = 40$	Simplify. Check your solution.

Example 2 **Solve $7y - 2y + 4 = 29$. Check your solution.**

$7y - 2y + 4 = 29$	Write the equation.
$5y + 4 = 29$	Combine like terms.
$\underline{-4 = -4}$	Subtraction Property of Equality
$5y = 25$	Simplify.
$\frac{5y}{5} = \frac{25}{5}$	Division Property of Equality
$y = 5$	Simplify. Check your solution.

More Two-Step Equations

Solving equations of the form $p(x + q) = r$, can be accomplished in one of two ways:

1. When p is a factor of the constant term, r, then:

- use the Division Property of Equality when p is a whole number or decimal.
- use the Multiplication Property of Equality when p is a fraction.

2. Regardless if p is or is not a factor of the constant term, r, you can use the Distributive Property.

Example 1 **Solve the equation $-3(x + 4) = 27$.**

$-3(x + 4) = 27$	Write the equation.
$\frac{-3(x+4)}{-3} = \frac{27}{-3}$	Division Property of Equality
$x + 4 = -9$	Simplify.
$x + 4 - 4 = -9 - 4$	Subtraction Property of Equality
$x = -13$	Simplify.

Example 2 **Solve $2(d - 10) = -6$.**

$2(d - 10) = -6$	Write the equation.
$2d - 20 = -6$	Distributive Property
$2d - 20 + 20 = -6 + 20$	Addition Property of Equality
$2d = 14$	Simplify.
$\frac{2d}{2} = \frac{14}{2}$	Division Property of Equality
$d = 7$	Simplify.

Solving Equations with Variables on Each Side

To solve equations with variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable on one side. Then solve the equation.

Example **Solve $12x - 3 = 4x + 13$.**

$12x - 3 = 4x + 13$	Write the equation.
$12x - 4x - 3 = 4x - 4x + 13$	Subtraction Property of Equality
$8x - 3 = 13$	Simplify.
$8x - 3 + 3 = 13 + 3$	Addition Property of Equality
$8x = 16$	Simplify.
$\frac{8x}{8} = \frac{16}{8}$	Division Property of Equality
$x = 2$	Simplify.

To check your solution, replace x with 2 in the original equation.

Check $12x - 3 = 4x + 13$	Write the equation.
$12(2) - 3 \stackrel{?}{=} 4(2) + 13$	Replace x with 2.
$24 - 3 \stackrel{?}{=} 8 + 13$	Simplify.
$21 = 21$ ✓	The statement is true.

Inequalities

A mathematical sentence that contains any of the symbols listed below is an **inequality**. The chart below will help you write inequalities.

<	>	≤	≥
• is less than • is fewer than	• is greater than • is more than • exceeds	• is less than or equal to • is no more than • is at most	• is greater than or equal to • is no less than • is at least

Inequalities can be graphed on a number line. This helps you see which values make the inequality true.

The direction of the line indicates whether numbers *greater than* or *less than* the number marked make the sentence true.

Example 1 **Write an inequality for the sentence.**

Fewer than 70 students attended the last dance.

Words	*Fewer than* 70 students attended the last dance.
Symbols	Let s = the number of students.
Inequality	$s < 70$

Solving Inequalities

Use the Addition and Subtraction Properties of Inequalities to solve inequalities. When you add or subtract a number from each side of an inequality, the inequality remains true.

Example 1 **Solve $12 + y > 20$. Check your solution.**

$12 + y > 20$	Write the inequality.
$12 - 12 + y > 20 - 12$	Subtraction Property of Inequality
$y > 8$	Simplify.

The solution is $y > 8$. You can check this solution by substituting a number greater than 8 into the inequality.

Use the Multiplication and Division Properties of Inequalities to solve inequalities.

- When you multiply or divide each side of an inequality by a positive number, the inequality remains true. The direction of the inequality sign does not change.
- For an inequality to remain true when multiplying or dividing each side of the inequality by a negative number, however, you must reverse the direction of the inequality symbol.

Example 2 **Solve $\frac{y}{-12} < 4$. Check your solution.**

$\frac{y}{-12} < 4$	Write the inequality.
$-12\left(\frac{y}{-12}\right) > -12(4)$	Multiplication Property of Inequality
$y > -48$	Simplify.

The solution is $y > -48$. You can check this solution by substituting a number greater than –48 into the inequality.

Solving Multi-Step Equations and Inequalities

Equations with grouping symbols can be solved by first using the Distributive Property to remove the grouping symbols.

Example 1 **Solve $2(6m - 1) = 8m$. Check your solution.**

$2(6m - 1) = 8m$	Write the equation.
$12m - 2 = 8m$	Distributive Property
$12m - 12m - 2 = 8m - 12m$	Subtraction Property of Equality
$-2 = -4m$	Simplify.
$\frac{-2}{-4} = \frac{-4m}{-4}$	Division Property of Equality
$\frac{1}{2} = m$	Simplify.

Check $2(6m - 1) = 8m$ Write the equation.

$2\left[6\left(\frac{1}{2}\right) - 1\right] \stackrel{?}{=} 8\left(\frac{1}{2}\right)$	Replace m with $\frac{1}{2}$.
$2(3 - 1) \stackrel{?}{=} 4$	Simplify.
$4 = 4$ ✓	The solution checks.

Some equations have no solution. The solution set is the **null** or **empty set,** which is represented by Ø. Other equations have every number as a solution. Such an equation is called an **identity.**

Example 2 **Solve each equation.**

a. $\mathbf{2(x - 1) = 4 + 2x}$

$$2x - 2 = 4 + 2x$$

$$2x - 2x - 2 = 4 + 2x - 2x$$

$$-2 = 4$$

The statement $-2 = 4$ is *never* true. The equation has no solutions and the solution set is Ø.

b. $\mathbf{-2(x - 1) = 2 - 2x}$

$$-2x + 2 = 2 - 2x$$

$$-2x + 2 - 2 = 2 - 2 - 2x$$

$$-2x = -2x$$

$$x = x$$

The statement $x = x$ is *always* true. The equation is an identity and the solution set is all numbers.